

**Time: 3 Hours****Max Marks: 60**

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

**UNIT-I**

1. If the temperature of the air is  $30^{\circ}\text{C}$  and the body cools from  $80^{\circ}\text{C}$  to  $60^{\circ}\text{C}$  in 12 minutes, then find the temperature of the body after 24 minutes. 10 M

**(OR)**

2. a) Solve  $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0$ . 5 M
- b) Find the orthogonal trajectories of the family of parabolas  $y^2 = 4ax$ , where 'a' is the parameter. 5 M

**UNIT-II**

3. Solve  $(D^2 + 5D + 6)y = e^{-2x} + \sin 2x$ . 10 M

**(OR)**

4. Solve  $(D^2 - 1)y = e^x + x^2e^x$ . 10 M

**UNIT-III**

5. a) Find the Fourier series representation of  $f(x) = x^2$  in the interval  $(-\pi, \pi)$ . 5 M

- b) Find the half – range Fourier cosine series of  $f(x) = x$  in the interval  $(0, 2)$ . 5 M

**(OR)**

6. Find the Fourier series representation of  $f(x) = e^{-x}$  in the interval  $(0, 2\pi)$ . 10 M

**UNIT-IV**

7. Find the maximum and minimum values of  $f(x, y) = x^3 + y^3 - 3axy$ . 10 M

**(OR)**

8. a) Find the Taylor's series representation of  $f(x, y) = x^4 + y^3x^2 + y^2$  at the point  $(1, 1)$ . 5 M

- b) Find the dimensions of the rectangular box open at the top of maximum capacity whose surface is 432 sq.cm. 5 M

**UNIT-V**

9. Form the Partial Differential Equation by eliminating arbitrary function  $f$  from  $f(x^2 + y^2, z - xy) = 0$ . 10 M

**(OR)**

10. a) Form the Partial Differential Equation by eliminating arbitrary constants  $a, b$  from  $(x - a)^2 + (y - b)^2 + z^2 = c^2$ . 5 M

- b) Solve  $(y + z)p - (z + x)q = x - y$ . 5 M

**UNIT-VI**

11. Find all the possible solutions of one-dimensional heat equation  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ . 10 M

**(OR)**

12. Solve the Partial Differential Equation  $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$  by using the method of separation of variables. 10 M

Time: 3 Hours

Max Marks: 70

Answer ONE Question from each Unit

All Questions Carry Equal Marks

All parts of the Question must be answered at one place

UNIT-I

- |       |                                                                                                                                                                   | Marks | CO | Blooms Level |
|-------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----|--------------|
| 1. a) | Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$                                                                                                                          | 7     | 1  | K3           |
| b)    | Solve $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$                                                                                                                        | 7     | 1  | K3           |
| (OR)  |                                                                                                                                                                   |       |    |              |
| 2.    | If the air is maintained at 30°C and the temperature of the body cools from 80° C to 60° C in 12 minutes, determine the temperature of the body after 24 minutes. | 14    | 1  | K3           |

UNIT-II

- |      |                                                                                   |    |   |    |
|------|-----------------------------------------------------------------------------------|----|---|----|
| 3.   | Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$                       | 14 | 2 | K3 |
| (OR) |                                                                                   |    |   |    |
| 4.   | Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 4y = \tan 2x$ | 14 | 2 | K3 |

UNIT-III

- |      |                                                               |    |   |    |
|------|---------------------------------------------------------------|----|---|----|
| 5.   | Solve the equation $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ . | 14 | 3 | K3 |
| (OR) |                                                               |    |   |    |
| 6.   | Solve $(D^2 - DD') = \cos x \cos 2y$                          | 14 | 3 | K3 |

UNIT-IV

- |       |                                                                                                        |   |   |    |
|-------|--------------------------------------------------------------------------------------------------------|---|---|----|
| 7. a) | What is the greatest rate of increase of $u = x^2 + yz^2$ at the point $(1, -1, 3)$                    | 7 | 4 | K3 |
| b)    | Find the angle between the normal to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$ | 7 | 4 | K3 |

(OR)

- |       |                                                                                             |   |   |    |
|-------|---------------------------------------------------------------------------------------------|---|---|----|
| 8. a) | If $V = \frac{xi+yj+zk}{\sqrt{x^2+y^2+z^2}}$ then show that $\nabla \times V = 0$           | 7 | 4 | K3 |
| b)    | Find $\text{curl } \vec{F}$ , where $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$ | 7 | 4 | K3 |

UNIT-V

- |      |                                                                                                                                                                              |    |   |    |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|---|----|
| 9.   | Verify Green's theorem for $\oint (xy + y^2)dx + (x^2)dy$ where $C$ is the curve bounded by $y = x^2, y = x$                                                                 | 14 | 5 | K3 |
| (OR) |                                                                                                                                                                              |    |   |    |
| 10.  | Verify Divergence theorem for the vector $\vec{F} = (4xz)\vec{i} - (y^2)\vec{j} + (yz)\vec{k}$ over the box bounded by the planes $x = 0, x = 1; y = 0, y = 1; z = 0, z = 1$ | 14 | 5 | K3 |

Time: 3 Hours

Max Marks: 70

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| <u>UNIT-I</u>   |                                                                                                                                                                                                                                                                                                                                                        | Marks | CO | Blooms Level |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----|--------------|
| 1. a)           | Solve $(1 - x^2) \frac{dy}{dx} - xy = 1$                                                                                                                                                                                                                                                                                                               | 7 M   | 1  | K3           |
| b)              | If the temperature of the air is $30^{\circ}\text{C}$ and the substance cools from $100^{\circ}\text{C}$ to $70^{\circ}\text{C}$ in 15 minutes, find when the temperature will be $40^{\circ}\text{C}$ .                                                                                                                                               | 7 M   | 1  | K3           |
| (OR)            |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 2. a)           | Solve $r \sin \theta - \cos \theta \frac{dr}{d\theta} = r^2$                                                                                                                                                                                                                                                                                           | 7 M   | 1  | K3           |
| b)              | When a switch is closed in a circuit containing a battery $E$ , a resistance $R$ and an inductance $L$ , the current $i$ builds up at a rate given by $\frac{di}{dt} + Ri = E$ . Find $i$ as a function of $t$ , how long will it be before the current has reached one-half of its final value is $E = 60\text{Hz}, R = 100\Omega, L = 0.1\text{H}$ . | 7 M   | 1  | K3           |
| <u>UNIT-II</u>  |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 3.              | Solve $(D^2 + 4D + 3)y = e^{-x} \sin x + xe^{3x}$                                                                                                                                                                                                                                                                                                      | 14M   | 2  | K3           |
| (OR)            |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 4.              | Solve $y'' + y = x \sin x$ by method of variation of parameters                                                                                                                                                                                                                                                                                        | 14M   | 2  | K3           |
| <u>UNIT-III</u> |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 5. a)           | Form the partial differential equation by Eliminating the arbitrary constants $a$ and $b$ from $z = a \log\left(\frac{b(y-1)}{1-x}\right)$                                                                                                                                                                                                             | 7 M   | 3  | K3           |
| b)              | Find the partial differential equation whose solution is $z = f(x^2 + y^2)$ where $f$ is any arbitrary differential function                                                                                                                                                                                                                           | 7 M   | 3  | K3           |
| (OR)            |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 6.              | Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$                                                                                                                                                                                                                                                                                                         | 14M   | 3  | K3           |
| <u>UNIT-IV</u>  |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 7. a)           | Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$                                                                                                                                                                                                                                            | 7 M   | 4  | K3           |
| b)              | Show that $(-x^2 + yz)\vec{i} + (4y - z^2x)\vec{j} + (2xz - 4z)\vec{k}$ is solenoidal                                                                                                                                                                                                                                                                  | 7 M   | 4  | K3           |
| (OR)            |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 8.              | If $\vec{a}$ is a constant vector, prove that $\text{Curl} \left( \frac{\vec{a} \times \vec{r}}{r^3} \right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5} (\vec{a} \cdot \vec{r})$                                                                                                                                                                    | 14M   | 4  | K3           |
| <u>UNIT-V</u>   |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 9.              | Verify Green's theorem for $\int_C x^2 y dx + x^2 dy$ where $C$ is a boundary of the triangle with vertices $(0, 0)$ , $(1, 0)$ and $(1, 1)$                                                                                                                                                                                                           | 14M   | 5  | K3           |
| (OR)            |                                                                                                                                                                                                                                                                                                                                                        |       |    |              |
| 10.             | Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$ .                                                                                                                                                                                                     | 14M   | 5  | K3           |